

# Conference

## “Infinite measure dynamics”

June, 6th to 9th 2017, UBO, Brest, France.

This document is also available in latex, html and pdf format.

### Schedule

Tuesday 6	Wednesday 7	Thursday 8	Friday 9
9h30-10h Reception	9h-10h R. Zweimüller	9h-10h J. Chaika	9h-10h R. Dougall
10h-11h E. Roy	10h-10h30 Break	10h-10h30 Break	10h-10h30 Break
11h-12h S. Brofferio	10h30-11h30 J. Aaronson	10h30-11h30 M. Bromberg	10h30-11h30 D. Thomine
		11h30-12h30 A. Málaga S.	
University canteen	Water tower	University canteen	University canteen
14h-15h F. Dal’bo			13h30-14h30 O. Glorieux
15h-16h D. Terhesiu	Walk by the sea	14h30-15h30 D. Dolgopyat	14h30-15h30 S. Gouëzel
16h-16h30 Break	Walk by the sea	15h30-16h Break	
16h30-17h30 (1)	Walk by the sea	16h-17h M. Stadlbauer	
		Restaurant	

<sup>1</sup> Short talks by young researchers.

### Speakers

Jon Aaronson (Univ. Tel Aviv)  
 Michael Bromberg (Univ. Bristol)  
 Françoise Dal’bo (Univ. Rennes 1)  
 Rhiannon Dougall (Univ. Warwick)  
 Sébastien Gouëzel (Univ. Nantes)  
 Emmanuel Roy (Univ. Paris 13)  
 Dalia Terhesiu (Univ. Vienna)  
 Roland Zweimüller (Univ. Vienna)

Sara Brofferio (Univ. Paris Sud)  
 Jon Chaika (Univ. Utah)  
 Dmitri Dolgopyat (Univ. Maryland)  
 Olivier Glorieux (IMPA)  
 Alba Málaga Sabogal (Univ. Paris 8)  
 Manuel Stadlbauer (Univ. Rio de Janeiro)  
 Damien Thomine (Univ. Paris Sud)

## Practical informations

The conference takes place in Brest, France. The city is located by the sea, the air temperature is between 18 and 28 Celsius degree, the water temperature is between 16 and 18 degree and there are no mosquitos. The beaches are located outside the city and are better reached by bus/car, a trip is planned on wednesday afternoon.

### Direction to the university

Concerning transportation, tickets can be purchased at stations and in buses but not in tramways. The same ticket works for buses, tramways and telepherique (1 hour duration) and must be stamped each time you enter a vehicle.

– *From the airport*

The airport is located North-East of the city. Either take a taxi (15 mn, 30 euros) or take the shuttle in front of the airport. Shuttle tickets (1.5 euros) are purchased in the shuttle. The shuttle goes to the tramway terminus station (10 mn) or to the railway station. The tramway goes to the center of the town (30 mn). Get off at **place de la liberté**. This is the city center, from which you can reach the university or your hotel in at most 15 minutes, either by bus or by foot.

– *From the city center*

The university can be reached from the town center or the railway station by bus (15mn) or by foot (30mn). Take the bus **number 1** either in front of the railway station or from **place de la liberté**. Tickets can be bought on the bus. Wait until the bus takes a huge blue bridge and get off at **Bouguen** close to **entry A** of the faculté des Sciences, just after the roundabout. A detailed map of the bus stations is available on the website of the Bibus bus company.

– *On the campus*

The mathematics department is located on third floor of the **bâtiment C**, the building closest to entry A. The secretary is in the library, close to the restroom and the coffee machine. The talks take place in **Amphitheater E**.

### Contact

Annick Nicolle is the secretary of the research laboratory.

email: annick.nicolle@univ-brest.fr, phone number: +33 (0) 2 98 01 62 07.

The local organizers are Yves Coudène and Françoise Pène.

Brest airport	<a href="http://www.brest.aeroport.fr">http://www.brest.aeroport.fr</a>
Airport shuttle	<a href="http://www.brest.aeroport.bzh/en/access-and-parking/airport-shuttle-bus">http://www.brest.aeroport.bzh/en/access-and-parking/airport-shuttle-bus</a>
Railway (SNCF)	<a href="http://www.voyages-sncf.com/">http://www.voyages-sncf.com/</a>
Brest Bus	<a href="http://www.bibus.fr">http://www.bibus.fr</a>
University location	<a href="https://www.univ-brest.fr/ufr-sciences/menu/Presentation/Venir-a-Brest">https://www.univ-brest.fr/ufr-sciences/menu/Presentation/Venir-a-Brest</a>
Math department	<a href="http://www.lmba-math.fr/laboratoire.html">http://www.lmba-math.fr/laboratoire.html</a>

## Participants

Jon Aaronson (Univ. Tel Aviv)  
Kamel Belarif (UBO)  
Adrien Boulanger (Univ. Paris 6)  
Sara Brofferio (Univ. Paris Sud)  
Michael Bromberg (Univ. Bristol)  
Jon Chaika (Univ. Utah)  
Jean-Pierre Conze (Univ. Rennes)  
Yves Coudène (UBO)  
Gilles Courtois (UPMC)  
Françoise Dal'bo (Univ. Rennes 1)  
Nguyen Thi Dang (Univ. Rennes 1)  
Jean-Marc Derrien (UBO)  
Yves Derriennic (UBO)  
Aziz El Hadji Abdou Diop (Dakar)  
Dmitry Dolgopyat (Univ. Maryland)  
Rhiannon Dougall (Univ. Warwick)  
Olivier Glorieux (IMPA)  
Sébastien Gouëzel (Univ. Nantes)  
Yves Guivarc'h (Univ. Rennes 1)  
Victor Kleptsyn (Univ. Rennes 1)  
Lingmin Liao (Univ. Paris-Est Créteil)  
François Maucourant (Univ. Rennes 1)  
Thi Hien Nguyen (UBO)  
Jean-Claude Picaud (Univ. Tours)  
Françoise Pène (UBO)  
Anja Randecker (Univ. Toronto)  
Emmanuel Roy (Univ. Paris 13)  
Alba Málaga Sabogal (Univ. Paris 8)  
Adamou Saidou (Univ. Dan Dicko Dankoulodo de Maradi)  
Benoit Saussol (UBO)  
Barbara Schapira (Univ. Rennes 1)  
Cesar Silva (Williams College)  
Manuel Stadlbauer (Univ. Rio de Janeiro)  
Samuel Tapie (Univ. Nantes)  
Dalia Terhesiu (Univ. Vienna)  
Damien Thomine (Univ. Paris Sud)  
Nasab Yassine (UBO)  
Hisatoshi Yuasa (Univ. Osaka Kyoiku)  
Roland Zweimüller (Univ. Vienna)

## **Titles and abstracts of the talks**

### **Jon Aaronson**

*Rational ergodicity properties and distributional limits of infinite ergodic transformations.*

In infinite ergodic theory, various weak and distributional limits replace the absolutely normalized pointwise ergodic theorem. We'll review the subject and then see that every random variable on the positive reals occurs as the distributional limit of some infinite ergodic transformation. As a corollary, we obtain a complete classification of the possible "A-rational ergodicity properties" for an infinite ergodic transformation.

The main construction follows by "inversion" from a cutting and stacking construction showing that every random variable on the positive reals occurs as the distributional limit of the partial sums of some positive, ergodic stationary process normalized by a 1-regularly varying normalizing sequence (indeed, here the process can be chosen over any EPPT).

Joint work with Benjamin Weiss. See arXiv:1604.03218

### **Sara Brofferio**

*On unbounded invariant measures of stochastic dynamical systems*

We consider stochastic dynamical systems  $X_n = \Psi_n(X_{n-1})$ , where  $\Psi_n$  are i.i.d. random continuous transformations of  $\mathbf{R}$ . We assume that  $\Psi_n(x)$  behave asymptotically like  $A_n x$ , for some random positive number  $A_n$ . The main example is the stochastic affine recursion  $X_n = A_n X_{n-1} + B_n$ , but this class includes other interesting processes such as reflecting random walks or branching process. Our aim is to describe invariant Radon measures of the process  $\{X_n\}$  in the critical case, when  $\mathbf{E} \log A = 0$ . Under optimal assumptions, we prove that those measures behave at infinity like  $dx/x$ . In the proof we strongly use some properties of random walks on the affine group. The talk will be based on a joint paper with Dariusz Buraczewski.

### **Michael Bromberg**

*Temporal distributional limit theorem for cocycles over rotations*

For a measure preserving system  $(X, \mathcal{B}, \mu, T)$  and a real valued function  $f$  on  $X$ , temporal random variables along an orbit of a fixed point  $x$  in  $X$  are obtained by considering the Birkhoff sums  $S_n(f, x)$ ,  $n = 1, \dots, N$  and choosing  $n$  randomly uniformly from  $1, \dots, N$ . These r.v.'s, measure the fraction of time that Birkhoff sums spend in various sets. If, under proper normalization, as  $N$  tends to infinity, these variables converge to a non-atomic distribution, we say that  $f$  satisfies a temporal limit theorem along the orbit of  $x$  (when the limit is Gaussian, we refer to this as temporal CLT). The aim of the talk is to introduce the relevant concepts and sketch a proof of a temporal CLT for piecewise constant cocycles with a single breakpoint, over an irrational rotation with a badly approximable rotation number. This result generalises earlier results by J.Beck and by D.Dolgopyat and O.Sarig. This is joint work with C.Ulcigrai.

## **Jon Chaika**

*Ergodicity of typical skew products over some interval exchange transformations*

Let  $T$  be a linear recurrent interval exchange transformation. This is a measure zero, but full Hausdorff dimension set of interval exchange transformations that are analogous to badly approximable rotations. We show that an  $\mathbb{R}$  valued skew product over such an IET by an integral 0 function that is a linear combination of characteristic functions of intervals is typically ergodic. Relevant terms will be defined. This is joint work with Donald Robertson.

## **Françoise Dal'bo**

*An example of a nonuniform lattice with infinite Bowen-Margulis measure*

Joint work with M. Peigné, J-C Picaud, A. Sambusetti.

I will explain how to construct a noncompact negatively curved Riemannian surface with finite volume admitting an infinite Bowen-Margulis measure.

## **Dmitry Dolgopyat**

*On Local Limit Theorems for hyperbolic flows*

I describe an approach to proving local limit theorems and related for flows based on (multidimensional) local limit theorem for associated Poincare map. Both finite and infinite measure case will be discussed. Based on a joint work with Peter Nandori.

## **Rhiannon Dougall**

*Growth of closed geodesics for infinite covers*

We are interested in the dynamics of the geodesic flow for infinite volume manifolds  $M$  which arise as a regular cover of a fixed compact (or convex cocompact) negatively curved manifold  $M_0$ . Writing  $h_M$  for the exponential growth rate of closed geodesics in  $M$ , we have that  $h_M \leq h_0$ , where  $h_0$  is the topological entropy of the geodesic flow for  $M_0$ . We answer the question of when there is a uniform gap  $h_M < h_0$  in  $M$  in terms of the permutation representations given by the covering  $M$  of  $M_0$ . The proof uses the symbolic dynamics for the flow, and so we formulate the analogous statements for countable state shifts obtained as group extensions of a finite state shift.

## **Olivier Glorieux**

*Hausdorff dimension and critical exponent of Quasi-Fuchsian Anti-de Sitter manifolds*

The aim of my talk will be to explain how classical invariants and theorems for groups acting on the hyperbolic space, can be extended to the Anti-de Sitter (AdS) setting. We will recall the notion of critical exponent and Hausdorff dimension for discrete action on the hyperbolic space and explain how we can define similar notions for a certain type of groups acting on AdS manifolds. We will finally explain how to get a rigid bound for these invariants in dimension 3 which is a result equivalent a famous result obtained by R. Bowen in '79. This is a joint work with D. Monclair.

## Sébastien Gouëzel

### *Quantitative Pesin theory for subshifts of finite type*

In non-uniformly hyperbolic dynamics, Pesin sets are measurable sets where the dynamics is very well understood. However, their definition makes these sets hard to control in a quantitative way, even when the underlying dynamics is hyperbolic. We will explain why such a control is useful, and what kind of bounds we can obtain. Joint work with L. Stoyanov.

## Alba Málaga Sabogal

### *Generic Wind-Tree Dynamics*

The Wind-Tree is an example of a dynamical system that has a very simple description, while having a very rich dynamics. It's a particular case of a billiard: a particle (the wind) goes straight forward as long as it does not meet any obstacle and it bounces elastically at each obstacle met. There is an infinite number of square obstacles which are distributed irregularly all over the plane. The dynamics will strongly depend on the distribution of the obstacles. The different configurations live in a Baire space - we can thus ask what happens for a generic configuration (i.e. a configuration in a  $G_\delta$ -dense set). We found that generic Wind Tree dynamics is actually nice: minimal, ergodic and of infinite ergodic index in almost every direction. This is joint work with Serge Troubetzkoy.

## Emmanuel Roy

### *Ergodic splittings of Poisson processes*

If  $N$  denotes a Poisson process, a splitting of  $N$  is formed by two point processes  $N_1$  and  $N_2$  such that  $N = N_1 + N_2$ . If  $N_1$  and  $N_2$  are independent Poisson processes then the splitting is said to be Poisson and such a splitting is always available (We allow the possibility to enlarge the ambient probability space). In general, a splitting is not Poisson but the situation changes if we require that the distributions of the point processes are invariant by a common underlying map that acts at the level of each point of the processes. We will prove that if this map has infinite ergodic index, then a splitting is necessarily Poisson if the environment is ergodic. This is a work in progress, with Elise Janvresse and Thierry de la Rue.

## Manuel Stadlbauer

### *Graph extensions of Gibbs-Markov maps and amenability*

The aim of the talk is to relate a general notion of amenability of graphs with the probability of return of a random walk with stationary increments. That is, for a Markov map  $T : X \rightarrow X$  with embedded Gibbs-Markov structure (i.e.  $T$  is a tower over a Gibbs-Markov map with full branches) and  $\kappa$  a map from  $X$  to the automorphisms of the graph, we relate the decay of

$$\mu(\{x : \kappa(T^n(x)) \cdots \kappa(T(x)) \circ \kappa(x)(o) = o\})$$

with the amenability of the graph. It turns out that on the level of the embedded Gibbs-Markov structure, amenability is equivalent to spectral radius equal to 1 of the transfer operator of the graph extension. In particular, this generalizes results by Kesten, Day and Derriennic and Guivarc'h for random walks with independent increments. With respect to  $T$ , the relation is more intricate and requires additional assumptions. These results have canonical application to the geodesic flow on  $\mathbf{H}/G$ , where  $G$  is a subgroup of a finitely generated Fuchsian group.

This is joint work with Johannes Jaerisch (Shimane, Japan) and Elaine Rocha (Salvador, Brazil).

## **Dalia Terhesiu**

*Exploiting semistable laws for i.i.d. random variables*

We recall that semistable laws is a class of infinitely divisible laws, which complements the more well known stable laws. I will recall some main, previously established, results on necessary and sufficient conditions for the existence of semistable laws for i.i.d. random variables. I will report on work in progress with Peter Kevei which aims toward a complete understanding of a limit law for null recurrent renewal chains, assuming that the involved return function is in the domain of a semistable law (as such, no strict regular variation is required). Some analogies with the Darling Kac law will be discussed. If time remains, I will present some results of work in progress with Douglas Coates on semistable laws for interval intermittent maps.

## **Damien Thomine**

*Induction invariance, harmonic functions and applications*

Given a random walk on  $\mathbf{Z}^d$ , one may be interested in a large variety of questions on its statistical properties (such as "What is the probability of being at a given site at a given time?"). Here, I shall discuss questions such as:

- Starting from 0, what is the probability of hitting site  $p$  before going back to 0?
- Starting from 0, what is the probability of hitting site  $p$  before site  $q$ ?

In the setting of random walks, the answer to these questions is well known, and involves the induction invariance of the solutions of the Poisson equation.

In the setting of  $\mathbf{Z}^d$  extensions of dynamical systems, however we lose the Markov property, and thus we cannot use these tools. However, I'll show that these can be partially replaced by a use of Green-Kubo's formula, which still satisfies some induction invariance in this more general setting. This gives us answers for dynamical systems such as the geodesic flow on periodic hyperbolic surfaces, or (in part) Lorentz' gases. Joint work with F. Pène (University of Brest).

## **Roland Zweimüller**

*Return- and hitting-time distributions of small sets.*

I will present some work on the asymptotics of return- and hitting-time distributions of small sets in certain infinite measure preserving systems, as the measure of these

sets decreases to zero (“rare events”). My focus will be on fairly nice concrete systems and an abstract setup accommodating them. This includes joint work with F. Pene, B. Saussol, and S. Rechberger.

## **Sponsors**

The conference is funded by the following agencies:

- l’université de Bretagne-Occidentale,
- l’université Bretagne-Loire et la région Bretagne,
- le laboratoire de mathématiques de Bretagne Atlantique et le CNRS,
- le groupement de recherche Platon,
- l’institut universitaire de France,
- le labex Lebesgue.

Yves Coudène, June 3, 2017