Martingale approximations for G-normal distribution and its applications

Bernard DE MEYER
PSE-CES, Université Paris 1, Paris, 75013, France
Xinpeng LI
School of Mathematics, Shandong University, Jinan, 250100, China
PSE-CES, Université Paris 1, Paris, 75013, France

Summary

The notion of G-expectation, which is a typical sublinear expectation, was firstly introduced by Peng. It is a powerful tool to deal with the volatility uncertainty problem in finance. Under this sublinear expectation framework, Peng developed the law of large numbers and central limit theorem under sublinear case, which indicate that G-normal distribution plays the same important role in the theory of sublinear expectations as normal distribution in the classical linear case.

The following representation theorem is a special case of Proposition 49 in Denis, Hu and Peng: If $\xi \sim \mathcal{N}(0, [\underline{\sigma}^2, \overline{\sigma}^2])$, then

$$\tilde{\mathbb{E}}[\varphi(\xi)] = \sup_{\theta \in \mathcal{A}_{[0,1]}([\underline{\sigma}, \overline{\sigma}])} E_P[\varphi(\int_0^1 \theta_s dW_s)],$$

where (W_t) is a Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$ and $\mathcal{A}_{[0,1]}([\underline{\sigma}, \overline{\sigma}])$ is the set of $[\underline{\sigma}, \overline{\sigma}]$-valued adapted process on an interval $[0, 1]$. In this presentation, we give another representation theorem by martingales in discrete time. This martingale approximation for G-normal distribution can be regard as a central limit theorem for martingales with variance uncertainty. As an application, we use it to calculate the maximum L^p-variation for martingales.