Persitent random walks. Convergence and application to insurance

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Let $(Y_n)_{n\geq 0}$ be a Markov chain valued in $\{-1, 1\}$. The associated persitent random walk is the process : $X_n := Y_0 + Y_1 + \cdots + Y_n$.
1) An application of persistent random walks to insurance is given. We also consider the validity of the model with real data.
2) Set $\alpha := P(Y_1 = 1|Y_0 = -1)$ and $\beta := P(Y_1 = -1|Y_0 = 1)$. Suppose :

$$\alpha = \alpha_0 + c_0\Delta_x, \quad \beta = \beta_0 + c_1\Delta_x,$$

with $\alpha_0, \beta_0 \in [0, 1]$, $c_0, c_1 \in \mathbb{R}$ et Δ_x is a small parameter ($\Delta_x \to 0$).
We are interested in the linear interpolation $(\widetilde{Z}_s^\Delta)_{s\geq 0}$ of $Z_s^\Delta := \Delta_x X_{s/\Delta_t}$ for any $s \in \Delta_t\mathbb{N}$.
Let $\rho_0 := 1 - \alpha_0 - \beta_0$. When $\rho_0 \neq 1$ and $\Delta_t = \big(\Delta_x\big)^2$, it is shown that there exists a renormalization of $(\widetilde{Z}_s^\Delta)_{s\geq 0}$ which converges in distribution to a Brownian motion with drift, as $\Delta_x \to 0$. In the case where $\rho_0 = 1$, we choose $\Delta_x = \Delta_t$, then $(\widetilde{Z}_s^\Delta)_{s\geq 0}$ converges to the zig-zag process.