Pricing Game Options with Call Protection: Doubly Reflected BSDEs with Call Protection and their Approximation

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In this work we consider the issue of numerical solution of a doubly reflected backward stochastic differential equation, with an upper barrier which is only active on random time intervals (doubly reflected BSDE with an intermittent upper barrier, RIBSDE for short).

From the point of view of financial interpretation, RIBSDEs arise as pricing equations of game options with call protection, in which the call times of the option's issuer are subject to constraints preventing the issuer from calling the bond on certain random time intervals. Moreover, in the standing example of *convertible bonds*, this protection is typically monitored in a possibly very path-dependent way. Calls may thus be allowed or not at a given time depending on the past values of the underlying asset, which leads, after extension of the state space to markovianize the problem, to highly-dimensional pricing problems. Deterministic pricing schemes are then ruled out by the curse of dimensionality, and simulation methods appear to be the only viable alternative.

The ***purpose of this work*** is to propose a practical and mathematically justified approach to the problem of solving numerically by simulation the RIBSDEs that arise as pricing equations of game options with call protection. Our ***main result*** establishes convergences rates for a discrete time approximation scheme by simulation to an RIBSDE.

The practical value of this scheme is then thoroughly assessed. For problems in dimension up to 30, the accuracy of the simulation scheme, in cases where alternative PDE results are available and can be used as a benchmark (problems with a high 'nominal' dimension, but endowed with a specific structure allowing one to reduce them to a low 'effective dimension'), typically lies in the range of one bp ($10^{-2}\%$) to 1% of relative error.

One thus gets a practical and mathematically justified approach to the problem of pricing by simulation convertible bonds with highly path-dependent call protection. More generally, this paper is an illustration of the real abilities of simulation/regression numerical schemes for high to very high-dimensional pricing problems.