On the Existence and Uniqueness of Solutions to Stochastic Differential Equations Driven by G-Brownian Motion with Integral-Lipschitz Coefficients

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Jan. 31st, 2010

Abstract

The objective of the work is to study the existence and uniqueness of solutions to stochastic differential equations (SDEs) and backward stochastic differential (BSDEs) driven by G-Brownian motion with integral-Lipschitz coefficients in the framework of sublinear expectation spaces.

On a well-defined G-expectation space, the following SDE driven by G-Brownian motion is considered:

$$X(t) = x + \int_0^t b(s, X(s))ds + \int_0^t h(s, X(s))d\langle B, B\rangle_s + \int_0^t \sigma(s, X(s))dB_s, \tag{1}$$

where $t \in [0, T]$ and the initial condition $x \in \mathbb{R}^n$ is given.

It is well known that under a Lipschitz condition on the coefficients b, h and σ, the solvability of (1) has been obtained by Peng (2007) and Gao (2009).

On the other hand, we establish the existence and uniqueness of the solution to (1) under the following so-called integral-Lipschitz condition:

$$|b(t, x_1) - b(t, x_2)|^2 + |h(t, x_1) - h(t, x_2)|^2 + |\sigma(t, x_1) - \sigma(t, x_2)|^2 \leq \rho(|x_1 - x_2|^2), \tag{2}$$

where $\rho : (0, +\infty) \to (0, +\infty)$ is a continuous, increasing, concave function satisfying

$$\rho(0+) = 0, \ \int_0^1 \frac{dr}{\rho(r)} = +\infty.$$

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Furthermore, we establish the existence and uniqueness of the solution to (1) under a "weaker" condition on b and h, i.e.,

$$|b(t,x_1) - b(t,x_2)| \leq \rho(|x_1 - x_2|);\ |h(t,x_1) - h(t,x_2)| \leq \rho(|x_1 - x_2|). \tag{3}$$

At the end of the work, we also consider the following type of G-BSDE:

$$Y_t = \mathbb{E}[\xi + \int_t^T f(s,Y_s)ds + \int_t^T g(s,Y_s)d\langle B,B\rangle_s|\mathcal{F}_t], \tag{4}$$

where $t \in [0,T]$ and $\xi \in L_G^2(\mathcal{F}_T;\mathbb{R}^n)$. Under the integral-Lipschitz condition:

$$|g(s,y_1) - g(s,y_2)|^2 + |f(s,y_1) - f(s,y_2)|^2 \leq \rho(|y_1 - y_2|^2),$$

the existence and uniqueness of the solution to (4) are obtained as well.