K. Kurdyka P. Orro : ***Algebraic geometry and sub-riemannian gradiant***

The objective of this course is to give some results on the behavior of the trajectories of the sub-riemannian gradient.
In this context the Lojasiewicz's inequality is not valid, and a trajectory of the horizontal gradient can be of infinite length, and can eventually accumulate on a closed curve.

We shall explain that, nevertheless, this behavior is exceptional: for a generic function the trajectories of its horizontal gradient have similar properties as in the case of the riemannian gradient.

In this way, we shall give elements of real algebraic geometry, then we shall recall the necessary results on genericity. The last partof the course will study the generic trajectories.

1. Semi-algebraic elements of the set theory:
Semi-algebraic sets: basic properties. Theorem of Tarski-Seidenberg.
Cellulary decomposition of semi-algebraic.

2. Topological and metric properties.
Theorem of local triviality
Lojasiewicz's inequality

3. Sard theorem, transversality in parameters, genericity of the functions of Walrus.
restatement of differential geometry. Sard Theorem.
Lemma of Walrus, passage of a critical point.
Theorem of Thom.

4. Trajectories of the sub-riemannian gradient of polynomial functions
sub-riemannian Structure which split. Horizontal gradient.
Horizontal critical points.
Behavior of generic trajectories of the horizontal gradient.

<u>References</u> :

M. Hirsch, Differential topology. Springer (1976)
J. Bochnak, M. Coste, M-F. Roy. G´eom´etrie semi-algébrique réelle. Springer (1987)
Subriemannian Geometry. Progress in Mathematics. Vol 144. Birkhäuser (1996)
Gradient horizontal de fonctions polynomiales, Annales de l'institut Fourier, 59 no. 5 (2009)
horizontal de fonctions polynomiales, Annales de l'institut Fourier, 59 no. 5 (2009)