P. Briand : hyppoelliptic operators, stochastic methods and sub-riemannian geometry

The purpose of this course is to give an illustration, on simple examples, the relations between probability theory, riemannian geometry and sub-riemannian geometry. Since results of Einstein, Feynman and Kac, it is well known that brownian movement gives rise to an alternativ approach for the study of the heat equation. In this way, the evolution of the heat kernel can be describe, in a precise way, by using probabilistic technics. This probabilistic method was taken again by G. Benarous in a more general context : the asymptotic comportment, when time tends to 0, of the heat kernel associated to an operator which satisfies Hörmander's conditions. This description uses the Carnot-Carathéodory metric associated to this problem. More recently, F. Baudoin and M. Bonnefont has been studied for the sub-lapacian in sub-riemannian geometry. We will present some of these results and also probabilistic tools used.