HIGHER CODIMENSION ISOPERIMETRIC PROBLEMS

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In this talk I will present a joint work with R. Mazzeo and F. Pacard on the construction of constant mean curvature submanifolds of arbitrary codimension in compact Rimannian manifolds with generic metrics. Our result is a generalisation of the theorem by R. Ye [8], which constructs families of CMC hypersurfaces which are small perturbations of geodesic spheres centered at nondegenerate critical points of the scalar curvature and a more recent paper of F. Pacard and X. Xu [6] where such hypersurfaces are obtained near critical points of another curvature invariant.

In arbitrary codimension, building on ideas of Almgren [1], we define CMC submanifolds to be boundaries of submanifolds which are critical points of a certain energy. Using the techniques introduced in [6], we construct such submanifolds near the nondegenrate critical points of a functional, which we call the partial scalar curvature and which is defined on the Grassmanian bundle of the ambient manifold and coincides with the scalar curvature in the case of codimension one.

References

[1] F. Almgren, *Optimal Isoperimetric inequalities*, Bulletin of the AMS, Vol. 13, 2, (1985), 123-126.

[2] U. Dierkes and S. Hildebrandt, and H. Lewy, *On the analyticity of minimal surfaces at movable boundaries of prescribed length.* J. Reine Angew. Math. 379 (1987), 100-114.

[3] D. Hoffman, *Surfaces in constant curvature manifolds with parallel mean curvature vector field.* Bull. Am. Math. Soc. 78, 247 (1972).

[4] H. B. Lawson, Jr., *Lectures on minimal submanifolds* , Berkeley CA. 1980

[5] F. Mahmoudi, R. Mazzeo and F. Pacard. Constant mean curvature hypersurfaces condensing along a submanifold. Geom. Funct. Anal. 16, no 4, (2006), 924-958

[6] F. Pacard et X. Xu. *Constant mean curvature spheres in Riemannian manifolds.* Manuscripta Math., 128 (3), 275-295, (2009)

[7] T.J. Willmore, *Riemannian geometry.* Clarendon Press (1997).

[8] R. Ye, *Foliation by constant mean curvature spheres*, Pacific J. Math. 147 (1991), no. 2, 381396.

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